

Exercise 7

Find an equation of the tangent line to the curve at the given point.

$$y = \sqrt{x}, \quad (1, 1)$$

Solution

Start by finding the slope of the tangent line to the curve at $x = 1$.

$$\begin{aligned} m &= \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1} \frac{\sqrt{x} - \sqrt{1}}{x - 1} \\ &= \lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1} \\ &= \lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{(\sqrt{x} + 1)(\sqrt{x} - 1)} \\ &= \lim_{x \rightarrow 1} \frac{1}{\sqrt{x} + 1} \\ &= \frac{1}{(1) + 1} \\ &= \frac{1}{2} \end{aligned}$$

The general equation of a line is

$$y = mx + b.$$

Here the slope is $m = 1/2$.

$$y = \frac{1}{2}x + b$$

Use the fact that the line passes through $(1, 1)$ to determine b .

$$1 = \frac{1}{2}(1) + b$$

$$1 = \frac{1}{2} + b$$

$$b = \frac{1}{2}$$

Therefore,

$$y = \frac{1}{2}x + \frac{1}{2}.$$

Below is a plot of the curve and the tangent line at $x = 1$.

